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Perception of Graduate Employability by Employers and Graduates in Kazakhstan and Worldwide

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Abstract: This paper is a brief overview of studies on perception of graduate employability by employers and graduates within the context of different regions with a particular focus on Kazakhstan. The research shows that employers worldwide emphasize soft skills rather than hard skills in graduates. Employers tend to assume that newly graduated employees lack soft skills while graduates, on the contrary, appear satisfied with their skills.
JEL classifications: I20, I21, I23
Keywords: Graduate Employability, Soft Skills, Hard Skills, Higher Education, Employers, Graduates

1. Introduction

During the last several decades, the concept of graduate employability has become popular worldwide. This paper tries to identify the graduate employability skills most desired by employers and students in Kazakhstan and worldwide. According to Succi and Canovi (2020), there is no mutual understanding among employers, higher education providers, and graduates on the importance of soft (transferable) skills, judging by the fact that graduates do not receive a sufficient set of general skills during their studies at the university. "[Employers and graduates] tend to operate in 'parallel universes' and it has been argued that young people are often not acquiring a sufficient portfolio of general skills during their university studies" (Mourshed, Patel, & Suder, 2014, as cited in Succi & Canovi, 2020, p. 4).

This study compares employers' and students' perceptions of the importance of graduate employability skills within the context of Kazakhstan, the United States of America, and European countries.

2. Soft and Hard Skills

Graduate employability competency consists of hard and soft skills. Hard skills are related to professional background and technical knowledge, while soft skills involve personal attitudes, habits, and perceptions of the world of employees. There are many definitions of soft skills. Knight and Page (2007), and Ciappei and Cinque (2014) as cited in Succi and Canovi (2020, p. 2) found "soft skills as "wicked competencies," as it is very difficult to define them since they can assume different forms within different contexts and keep developing along the entire lifetime." Succi and Canovi (2020) found that over the last five to 10 years, there is an increased emphasis on soft skills. In addition, companies consider soft skills to be more important for graduates.

The following sections explore and analyze both students' and employers' perceptions of soft and hard skills in the contexts of different regions.

3. The USA

Research published by the *Wall Street Journal* indicated that more than 90% of nearly 900 surveyed executives regarded soft skills as either equally or more important than technical skills, while 89% of the same respondents stated they have a "very or somewhat difficult" time finding employees possessing the required soft skills (US Chamber Foundation, 2017, p. 4).

Herren (2008), as cited in Williams (2015), states that employees in the United States have difficulty in applying their soft skills to their working practice. Moreover, Williams noted that "the same problem is also apparent in the local setting of the study, as the readiness for employment of students of a local community college [does] not meet employers' expectations" (Williams, 2015, p. 3). The results of the college work experience program showed that in the opinion of employers, "some of the students are technically competent, while some students' soft skills and competencies are far below expectations of prospective employers" (Williams, 2015, p. 3). Anderson et al. (1999), as cited in Williams (2015), confirmed that although technical training is also important, employers expect a multi-skilled workforce. According to Jaschik (2015), as cited in Schooley (2017), students believe that they graduated from college with all needed skills and are generally well prepared for their jobs. The results of the survey conducted by the Association of American Colleges and Universities indicate that "the students consistently ranked themselves prepared in areas where employers are ranking them as extremely unprepared" (Schooley, 2017). About 60% of students believe that they possess fitting soft skills expected by employers such as communication, critical thinking, being innovative, teamwork, and problem solving, while only 27% of employers believe that students are competent in the same skills.

4. Western Europe

Fifty-four percent of employers in the United Kingdom recognize graduates as highly qualified but lacking soft skills (Smith, 2015, as cited in Succi and Canovi, 2020).

The survey conducted by Succi and Canovi (2020) among employers and students in Germany, Italy, and other European countries in 2018 revealed that soft skills are ranked as the most important to enhance graduate employability. According to 60.2% of employers who participated in the survey, students are not well or very well prepared. Respondents noticed that graduates lacked self-awareness and the ability to identify personal strengths and weaknesses. Succi and Canovi (2020, p. 8) noted that "employers distinguished clearly between the level of preparation provided by business schools or by universities, which offer less exposure to the 'real world,' [than is] conveyed, for example, by internships, case studies, and corporate testimonials." Employers also emphasized graduates' unrealistic expectations and their lack of responsibility and interest in acquiring and developing soft skills. The results of the same survey indicated that graduates ranked soft skills as less important than employers did when compared to technical skills.

5. Eastern Europe

The results of a survey conducted in Poland show that the educational output of vocational schools in Poland do not meet the labor market's requirements (Wiśniewska & Wiśniewska, 2019). The report titled "The Balance of Human Capital" emphasized that "the issue of mismatched competencies is mainly related to soft competencies such as communication (oral and written), teamwork, adaptability, goal orientation, and influencing others" (Kocór, 2015 as cited in Wiśniewska and Wiśniewska, 2019, p. 212). The results of the survey show that regardless of the job position and size of enterprises, employers expect and value soft skills rather than hard skills.

Another study compared the expectations of Slovak employers and students in terms of employability skills. According to research conducted by Lisá, Hennelová, and Newman (2019), employers perceived a lack of appropriate skills as the biggest obstacle in hiring graduates. Another barrier was the non-acceptance of salary. This survey involved 27 different-sized companies from different segments and 534 students from Slovakia. The results show that employers and students similarly perceived the importance of skills. "The skill of engagement and willingness to do something extra was considered as more important by employers in comparison to student responses" (p. 71). Graduates overestimated the level of their skills compared to their employers' assessment. Salary expectations of students were higher than those offered by employers despite the fact that graduates lacked practical skills (Lisá, Hennelová, & Newman, 2019).

6. Kazakhstan

According to the survey conducted by the Boston Consulting Group and the online international recruitment company The Network, 61% of people from 197 countries (366,000 respondents) believe that global trends tend to influence their jobs. Many respondents said that they are willing to acquire new skills to become attractive candidates for different jobs (Forbes Kazakhstan, 2019). Results of the survey revealed two megatrends: Technology changes and globalization—and two possible reactions of people to these changes. One is the willingness of people to adapt new skills to their current jobs (upskilling) and another one is the desire of people to adapt new skills to different jobs (reskilling).

Attitudes toward upskilling and reskilling, and the ways to achieve them, varied from country to country, nationality, age, and education of respondents. These trends have significant implications for government, companies, and education. The survey identified that most people assume that they are supposed to be multifaceted to be both good collaborators and critical thinkers. Respondents from Kazakhstan devoted more time to their studies than the world average (71% versus 65%). However, only 61% of respondents from Kazakhstan expressed readiness for retraining and change of profession, compared with 67% globally (Forbes Kazakhstan, 2019). Also, results showed that our compatriots pay much more attention to self-study or on-the-job training than to study within the educational system. About 6,000 respondents from Kazakhstan participated in this survey and ranked skills by level of importance (Table 1).

Table 1
Skills Ranked by the Level of Importance by Respondents from Kazakhstan

Rank	Degree of importance	Skills
1, 2, 3	most important skills	analytical skills, complex problem-solving, leadership skills
4, 5, 6	most important skills	communication skills, creativity, innovation
7, 8, 9	less important	ability to cooperate, agile, critical thinking
10, 11, 12, 13	not important skills	ability to adapt, risk assessment, emotional intelligence, intercultural awareness

Source: Forbes Kazakhstan, 2019

The results of respondents from the Russian Federation coincide with those from Kazakhstanis in eight out of 13 indicators.

Emotional intelligence and flexibility or adaptability, which were ranked last by Kazakhstanis in 2019, have become among the top 10 skills that employers look for in the labor market. It is not surprising, since in our globalized economy it is vital to be flexible, to react quickly to rapid changes, and to have high emotional intelligence to overcome difficulties.

Another trend that should be mentioned is the desire of Kazakhstanis to study on their own outside of the education system. According to Olzhas Ordabayev, CEO of Atameken Chamber of Entrepreneurship, as cited in *Kursiv* (2019), 61 Kazakhstani higher education institutions (HEIs) produce future unemployed graduates. In other words, graduates do not meet the requirements of Kazakhstani employers.

Most Kazakhstani higher education institutions develop internal policies and provisions on professional practice for students (professional internships). As stated in the Law on Education of the Republic of Kazakhstan, the goals, objectives, and program of professional internships are determined by curriculum and catalog approved by educational organizations. For example, Regulations on Professional Practice of Students of the Abai Kazakh National Pedagogical University (2019) state that students are expected to improve their basic and special competencies during their pedagogical internships. In the list of competencies, the following soft skills are included: Self-confidence, positive attitude, adequate professional self-esteem, openness to accepting other positions and points of view, emotional stability, the ability to stay calm in difficult situations, and empathy. Such specific requirements could contribute to the development of soft skills of students. However, the effect of successfully passing professional internship on the development of soft skills in graduates has not, to my best knowledge, been thoroughly researched in the context of Kazakhstan. It could be an area for further investigation.

The current neoliberal regime supports and strengthens the voice of employers. So, students after graduation from universities are expected to meet employers' expectations and to monitor changing employability trends. Large companies are willing to hire specialists who are able to operate in a complex environment and easily adapt to changes. The labor market demands graduates with "ready for work" skills. Some authors state that "policymakers and employers consider HEIs as 'drivers of economic growth'" (Sin and Neave, 2016, as cited in Succi and Canovi, 2020, p. 4) while they have also criticized HEIs for not adequately preparing graduates for the labor market (Moreau and Leathwood, 2006; Tomlinson, 2012; Clarke, 2018, as cited in Succi and Canovi, 2020, p. 4). Other authors add that "scholars noted that changes such as increased globalization, greater job insecurity, massification of higher education, and the shift to a knowledge economy have led to the need for graduate employability" (Bauman, 2003; Sin & Neave, 2016; Clarke, 2018, as cited in Succi and Canovi, 2020, p. 2).

Unemployment can occur for such reasons as high competition and lack of jobs in the labor market, the lack of skills of graduates or employers, economic crisis, corruption, and nepotism. According to the survey conducted by Atameken, as cited in Baigenews portal (2019), the number of HEI graduates in late 2019 and early 2020 is 5.3 times higher than the demand in the labor market. Moreover, to meet the requirements of employers, Kazakhstani HEIs are expected to adopt their academic programs to produce highly demanded graduates, especially in new pandemic and post-pandemic realities.

7. Covid-19 Reality

Kazakhstanis understand that under the influence of technology and globalization, their work and life in general can change. Our compatriots are ready to take the time to self-study and develop their skills. However, did they prioritize skill preferences correctly? Is it the right choice to focus on development of hard skills while the rest of the world emphasizes soft skills?

The table below contains abstracts from different sources about skills that have become important during the Covid-19 pandemic (Table 2).

Table 2

Skills Gaining Importance During the Covid-19 Pandemic

"The Future of Jobs" report by World Economic Forum revealing the top 10 skills needed by 2020	Ten of the key skills employers are looking for	"The 10+ Most Important Job Skills Every Company Will Be Looking For In 2020"
1. Complex problem-solving	1. Communication skills	1. Data Literacy
2. Critical thinking	2. Flexibility	2. Critical Thinking
3. Creativity	3. Determination	3. Tech Savviness
4. People management	4. Teamwork	4. Adaptability and Flexibility
5. Coordinating with others	5. Ability to Learn	5. Creativity
6. Emotional intelligence	6. Problem-solving	6. Emotional Intelligence (EQ)
7. Judgment and decision-making	7. Loyalty	7. Cultural Intelligence and Diversity
8. Service orientation	8. Resilience	8. Leadership Skills
9. Negotiation	9. Self-discipline	9. Judgment and Complex Decision Making
10. Cognitive flexibility	10. Tech Skills	10. Collaboration

Sources: hrvisionevent.com, studyworkgrow.com.au, *Forbes*

A study published as a result of the World Economic Forum (2020) assumes that during and after the pandemic people should acquire two key skills: Learning to learn and learning to discern. The first one is the ability "to rapidly gain skills and knowledge to adapt to changes and succeed." The learn-to-discern approach helps people build awareness of their psychological interactions and emotional impulses. In other words, people should be able to recognize and resist manipulative content.

8. Recommendations

Many international companies have their own academies to prepare newcomers and fill the gaps in their knowledge and skills. HEIs also provide a spectrum of opportunities to develop soft skills through such venues as social clubs, obligatory industrial internships, and projects. Pace (2011, as cited in Williams, 2015, p. 32) found that "students and employers have differing views of what constitutes relevant soft skills; similarly, students and employers also view the degree of importance of some soft skills from different perspectives. Additionally, the nature of soft skills requirements depends on the industry."

Each employer demands different skills, and employers and students perceive the importance of soft and hard skills differently. So, it is impossible to develop one universal guideline for all stakeholders. The possible solution of this dilemma might be to focus on fundamental knowledge; in other words, to produce well-rounded graduates (Vedomosti, 2017). According to the Rektor of Tomsk State University, Galazhinskiy, as cited in Vedomosti (2017), such a person would have a worldview while maintaining the position of a researcher in relation to her own professional activity. Many elite universities that are financially independent of the state, such as Oxford University, take this position in preparing graduates. They do not focus on the needs of employers, but nevertheless their students are the ones most demanded in the labor market (Boden & Nedeva, 2010).

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9. Summary

English: This survey of the literature on graduate employability finds that employers worldwide stress soft skills rather than hard. Unlike employees, graduates appear to be satisfied with their job skills.

Russian: В этой статье представлен краткий обзор существующих исследований восприятия возможности трудоустройства выпускников работодателями и выпускниками в контексте различных регионов с особым акцентом на Республику Казахстан. Исследование показывает, что работодатели во всем мире склонны уделять больше внимания мягким навыкам, чем твердым навыкам выпускников. Работодатели склонны предполагать, что недавно окончившим обучение сотрудникам, как правило, не хватает мягких навыков, тогда как выпускники, напротив, удовлетворены уровнем своих навыков после окончания учебы.

Kazakh: Бұл мақалада жұмыс берушілер мен түлектердің жұмысқа орналасу қабілеттілігі тұжырымдамасын қабылдаулары туралы зерттеулерге Қазақстан Республикасына ерекше екпін қоя отырып әр түрлі аймақтардың контекстінде қысқаша шолу ұсынылған. Зерттеулер көрсеткендей, бүкіл әлемде жұмыс берушілер түлектердің тұрақты дағдыларына емес, икемді дағдыларына көп көңіл бөледі. Жұмыс берушілер арасында жаңадан бітірген қызметкерлерге икемді дағдылар жетіспейді деп ойлау тенденциясы байқалады, ал түлектер, керісінше, өздерінің оқу бітіргеннен кейінгі дағдыларының деңгейіне қанағаттанады.

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Attitudes of Undergraduates in Kazakhstan Toward Clinical Trials

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Abstract: Clinical trials are essential for discovering treatments and preventing the spread of preventable diseases. However, recruitment of participants for clinical studies sometimes faces obstacles due to an inadequate number of study participants. This study explores the attitudes of undergraduate students in Kazakhstan towards participating in clinical trials. The study administered 13-item questionnaires to 187 undergraduates in an English-speaking university in Kazakhstan that is based on the United States model of education. Of the 187 questionnaires, 162 were correctly completed and returned (a response rate of 86.6%). Results suggest that clinical research teams should endeavor to create universal themes in their recruitment strategies to strengthen positive attitudes among potential participants. The teams should also consider cultural differences when recruiting for clinical trials. The study provides insights into factors that should be considered in recruiting potential young clinical trial participants in this Central Asian country.

JEL codes: I10, I18, I19

Keywords: Clinical Trials, Attitudes, Motivations, Knowledge, Recruitment, Kazakhstan

1. Introduction

Clinical trials are important for developing treatment regimens and vaccines. They also represent a basic component of the contribution of national health systems towards eliminating preventable diseases (Pillay & Wassenaar, 2018). In fact, regulators require clinical studies in many countries (including the United States, Canada, the European Union, and Japan) for new drugs to be approved and marketed. Patient recruitment has been widely recognized as a major problem in completion of clinical studies (Huang et al., 2018).

Participants' decisions to volunteer for clinical research are affected by such factors as their age, motivations, cultural norms, and other internal and external variables (Friesen & Williams, 2016; Bruce et al., 2014). Trials can fail when too few patients are recruited, threatening the reliability and validity of findings. Furthermore, attitudes of patients in may vary from country to country due to differences in health literacy, cultural values, trust in physicians and the healthcare system, access to care, knowledge and experience about the clinical trial process, perceived risk factors, and perceived benefits (Abushouk et al., 2016).

Understanding patients' attitudes toward participation in clinical research is critical to the effective and ethical design of such studies (Dawson & Kass, 2005; Wu et al., 2015). This study surveys undergraduates in an English-speaking, American-styled university in Kazakhstan to explore their attitudes and perceptions towards participating in clinical trials. Results of this study offer insights into how to increase clinical trial participation in Kazakhstan.

In this paper, Section 2 reviews the literature on clinical trials. Section 3 discusses the methodology of this study. Section 4 presents the findings, and Section 5 discusses them. Section 6 identifies limitations of the study, while Section 7 concludes the paper.

2. Literature Review

Several studies have examined the attitudes and willingness of participants in clinical trials. With respect towards clinical trials, some of these studies focused on attitudes, perceptions, and willingness of adult citizens in the general public, patients, and students. A selection of these studies is discussed below.

In an exploratory study, Tohid et al. (2017) examined the perceptions and attitudes of adult residents of Qatar toward participating in clinical trials. Survey questionnaires were sent to 2,379 residents aged 18 and above. Results indicated that only 5.7% of participants (*n* = 136) had previously been approached to participate in a clinical research study. Of these, 63.4% (*n* = 86) had agreed to participate, while 36.6% (*n* = 50) had declined. Reasons given for declining participation included: Time constraints; lack of awareness about clinical research; and lack of interest. Reasons given for participating in clinical trials included: Helping others (altruism); and improving access to healthcare. Thus, there is a need for more studies into the factors that affect clinical research participation in order to improve recruitment strategies in the Middle Eastern country.

In a study of the awareness and perceptions of Canadian citizens of the benefits and risks of participating in clinical trials, Willison, Richards, Orth, Harris, and Marlin (2018) electronically surveyed 1,602 residents of Ontario and British Columbia. According to results, 68% of the respondents showed positive attitudes toward participating in clinical trials. However, 43% felt that they were not well-informed about various aspects of the clinical trials process, while 37% had no opinion regarding the benefits and risks of participating in such research.

Friesen and Williams (2016) investigated the attitudes and willingness of citizens of the United States to participate in dental clinical trials. They administered questionnaires to 176 adults in the Midwest of the country. They found that 80.9% of respondents stated that their decision to participate in the study was easy while 50.3% participated in the study in order to help others. The study also found that 78.2% of the participants were motivated by financial incentives.

On patients’ attitudes about clinical research, Moorcraft et al. (2016) examined the willingness and perceptions of cancer patients to participate in clinical trials in a hospital in the United Kingdom. Questionnaires were sent to 276 patients; 263 patients completed them (a response rate of 95%). The majority (88%) of respondents consented to being involved in a clinical trial. Factors that influenced patients’ decisions to participate included that it provided the best available treatment option, and that the trials could benefit others. Seventy-five percent of the respondents indicated that patients should be informed about trial results, while 50% of the participants wanted more information on the trial procedures.

Al-Tannir, El-Bakri, and Abu-Shasheen (2016) examined the attitudes and perceptions of Saudi patients toward clinical trials. Self-administered questionnaires were given to 232 Saudi hospital patients. The study found that 73.8% of respondents were willing to participate in clinical trials after consulting their personal doctors, while 58% would be motivated to participate even if they were healthy. Thirty-seven percent of the respondents believed that patients who participated in clinical studies receive better medical care.

Finally, Staniszewska et al. (2018) investigated the attitudes of patients toward clinical trials in Poland by administering questionnaires to 256 patients. The study found that 64.8% of the patients were willing to participate in clinical trials. Fifty-four percent of the participants would be willing to participate in the clinical trial even if they were healthy. In addition, 69.1% indicated fear of possible effects, while 12.9% stated that frequent hospital visits were barriers to participating in clinical research.

Since university students are potential opinion leaders in society, it is important to examine factors that affect their attitudes and perceptions about clinical studies. We turn to this question.

3. Methodology

The study used information from secondary sources and primary data. Secondary sources included published articles in academic journals as well as reports and policy documents from international organizations. These included the

- World Health Organization;
- Statistical Agency of Kazakhstan (https://stat.gov.kz/);
- World Bank (www.worldbank.org);
- United Nations Development Program (www.undp.org);
- UN Educational, Scientific and Cultural Organization (www.unesco.org);
- Ministry of Health of the Republic of Kazakhstan (https://www.gov.kz/memleket/entities/...); and
- European Observatory on Health Systems and Policies (www.euro.who.int/en/countries/kazakhstan).

Primary data were obtained from survey questionnaires (Yin, 2009; Saunders, Lewis, & Thornhill, 2003). A total of 187 questionnaires were distributed to the students, and 162 were correctly completed and returned (a response rate of 86.6%).

Convenience sampling was used to collect the data. Convenience sampling is a method of collecting samples whereby only people who are easily accessible to the researcher can be questioned (Edgar & Manz, 2017). It is an inexpensive way to gather data and does not require any pattern in selecting respondents as do other sampling methods**.** Questionnaire items were based on the Likert scale as follows: 1 = Strongly disagree; 2 = Disagree; 3 = Neutral; 4 = Agree; and 5 = Strongly agree. Appendix A shows a sample of the questionnaire. The survey was conducted between October and December 2019.

4. Findings

There were a total of 162 students in the survey; 65% were female students, and 35% male students. Table 1 shows their responses about attitudes and beliefs towards participating in clinical trials. We discuss these below.

Clinical trials are done to improve healthcare standards: Sixty-seven percent of the survey sample agree or strongly agree that clinical trials improve healthcare standards worldwide. According to Dawson and Kass (2005), improved health standards in terms of disease prevention and control, better treatment methods, etc. can only be achieved through human clinical trials. Such trials are therefore needed to address global health challenges (Walsh & Sheridan, 2016; Bohner & Wanke, 2002).

Clinical trials should be voluntary: According to Friesen and Williams (2016), public information by governments is necessary to educate and encourage the public about the need for human clinical trials. In our study, 67% of the respondents agree or strongly agree that people should not be forced to participate in clinical studies. Participation in clinical research should be voluntary once all accurate and relevant information are provided to participants.

My greatest concern about participating in clinical trials is potential risks to my health: There is always the likelihood that something may go wrong during the clinical trial and adversely affect the health of the participant (Diemert et. al., 2017; Huang et al., 2018). In our survey, 57% of the students agree or strongly agree that they are concerned that participating in clinical research might endanger their health. This suggests that clinical trial organizers need to do a better job in informing the public about the rigor and due care taken in selecting trial participants as a means of ensuring that minimal or no adverse consequences are experienced due to the trials.

Table 1
Responses on Attitudes and Beliefs Towards Clinical Trials

Factor	**Agree**		**Strongly agree**		**Total positive Responses Agree/Strongly agree**		**Disagree/strongly disagree**		**Neutral**	
	No.	**%**	**No.**	**%**	**No.**	**%**	**No.**	**%**	**No.**	**%**
Clinical trials are done to improve health standards	30	18	79	49	***109***	***67***	41	25	12	8
Clinical trials should be voluntary	38	23	71	44	***109***	***67***	16	10	37	23
My **greatest concern** in participating in clinical trials is the potential risk to my health	10	6	83	51	***93***	***57***	61	38	8	5
People should be paid to participate in clinical trials	28	17	61	38	***89***	***55***	54	33	19	12
I am **more likely** to participate in clinical trials if it will make or my loved one live longer	15	9	71	44	***86***	***53***	64	39	12	8
I am **more likely** to participate in clinical trials if doing so will help other patients (altruism)	23	14	58	36	***81***	***50***	69	42	12	8
People who participate in clinical trials should keep it a secret because of the stigma attached to it (e.g. being used as a guinea pig)	24	15	39	24	***63***	***39***	70	43	29	18
I am **less likely** to participate in clinical trials if my medical records will be made public	14	9	49	30	***63***	***39***	50	31	49	30
I am **more likely** to participate in clinical trials if advised to do so by my doctor	19	12	39	24	***58***	***36***	73	45	31	19
My **greatest concern** in participating in clinical trials is that I may not be able to withdraw if I change my mind in the middle of the trial	11	7	39	24	***50***	***31***	101	62	11	7
Clinical trials are costly for patients to participate	8	5	21	13	***29***	***18***	105	65	28	17
Clinical trials are used by pharmaceutical companies only to make money	13	8	11	7	***24***	***15***	120	74	18	11
Children should be able to participate in clinical trials	5	3	11	7	***16***	***10***	131	81	15	9

Source: Field data (2019).

People should be paid to participate in clinical trials: Financial incentives have been used to encourage participation in clinical studies (Wu et al., 2015; Pillay & Wassenaar, 2018; Staniszewska et al., 2018). In our study, 55% of the respondents agree or strongly agree that people should be paid to engage in clinical trials. This may be to compensate participants for time lost from work, for the opportunity cost of other activities that they should have been involved in, etc. Other studies, such as Huang et al. (2018); Jones, Braz, McBride, Roberts, & Platts-Mills (2016); and Walsh and Sheridan (2016), have shown that financial reward motivates participation in clinical trials.

I am more likely to participate in clinical trials if it will make me live longer: People are more likely to engage in clinical studies when they have an illness for which there is no provable cure. If patients are convinced that clinical trials will lead to treatments that will prolong their lives, they are more likely to participate in such research. Results of the survey show that 53% of the sample agree or strongly agree that they will participate in clinical studies if doing so would prolong their lives. This confirms the findings of other studies that found that the ability of clinical trials to prolong patients' lives can motivate participating in such studies (Bohner & Wanke, 2002; Fiedler & Bieber, 2013; Vittalrao, Kumari, Kumari, Gill, & Thomson, 2018).

I am more likely to participate in clinical trials if doing so will help other patients: Altruism has been found to be a motivating factor in clinical trial participation (Friesen & Williams, 2016; Staniszewska et al., 2018). This is especially important when it involves trials that can lead to potential treatment of loved ones. In our survey, 50% of the respondents agree or strongly agree that they would be willing to participate in clinical trial if it will help other patients.

People who participate in clinical trials should keep it a secret because of the stigma attached to it (e.g., being used as a guinea pig): The perception that participants in clinical studies are viewed as "guinea pigs" used for experiments can discourage participation (Staniszewska et al., 2018). Thirty-nine percent of the survey students agree or strongly agree that their participation in clinical trials should be kept a secret because of this perception. Governments and trial organizers should do more to change this perception through public education campaigns.

I am less likely to participate in clinical trials if my medical records will be made public: Confidentiality of medical information is a tenet of medical practice, and this applies to clinical trials (Willison et al., 2018). In this study, 39% of the students agree or strongly agree that they would not participate in clinical trials if their medical records were not kept secret. Lack of confidentiality of medical records has been found to be a barrier to participation in clinical trials by studies such as Al-Tannier et al. (2016) and Wu et al. (2015).

I am more likely to participate in clinical trials if advised to do so by my doctor: Patients trust their doctors especially if the doctor-patient relationship has existed for a long time (Kaplan et al., 2015). A doctor's influence on a patient's decision can be enhanced by their level of mutual trust. Results of the survey show that 36% of the sample agree or strongly agree that they would be more willing to participate in clinical studies if encouraged to do by their doctor.

My greatest concern about clinical trials is that I may not be able to withdraw if I change my mind in the middle of the trial: The belief that a participant in clinical studies may find it difficult to withdraw before the end of the trials can also be a barrier to participation (Dawson & Kass, 2005; Bruce et al., 2014). Even when it may be possible to withdraw before the end of the trial, participants may fear potential health implications that may arise when they are no longer under the observation of the clinical research team. In our survey, 31% of the sample agree or strongly agree that the fear of not being able to withdraw in the middle of trials and the accompanying potential complications may weaken the intention to participate. This finding confirms such studies as Bohner and Wanke (2002) and Bruce et al. (2014).

Clinical trials are costly for patients to participate in: The perceived costs of participating in clinical trials, such as time and inconvenience, can discourage potential participants. Consequently, 18% of the students surveyed agree or strongly agree that these costs pose a barrier to participation. Clinical research planners should try to mitigate the cost perceptions of participants (Abushouk et al., 2016; Wu et al., 2015).

Clinical trials are run by pharmaceutical companies only to make money: Due to lack of education about the clinical trial process and its benefits for society, some members of the public think that such trials are conducted by pharmaceutical firms solely to make money (Bohner & Wanke, 2002; Wu et al., 2015). Unfortunately, this view is shared by 15% of the study sample who agree or strongly agree that pharmaceutical firms engage in clinical trials solely for financial gains.

Children should be able to participate in clinical trials: The idea of using children in clinical research poses a dilemma since only their parents or guardians can give the required consent (Caldwell, Murphey, Butow, & Craig, 2004; Joseph, Craig, & Caldwell, 2015; Sammons & Starkey, 2012). On one hand, the potential long-term effects of the trials on the participants may not be known; but on the other hand, without clinical trials involving children, it would be difficult to develop treatments for childhood diseases. Results of this study shows that 10% of the students agree or strongly agree that children should participate in clinical research.

5. Discussion

The ability to recruit and retain study participants is a major concern for clinical trial organizers. This study focuses on the intention of undergraduates in Kazakhstan to participate in clinical trials. Overall, the highest positive responses of students relate to the ideas that clinical trials are needed to improve the standards of healthcare and should be voluntary (at 67%).

Safety was the greatest concern among the students (57%). More than half of the students also believe that people should be paid to participate (55%); and they are willing to participate if it would enable a loved one to live longer (53%). Curiously, only 50% of the students would participate in clinical trials if it would help other patients (altruism). In addition, only 10% of the respondents believe that children should be able to participate in clinical studies; 15% believe that pharmaceutical companies conduct clinical trials to profit; and 31% of the respondents are concerned that they may not be able to withdraw from a trial if they change their minds in the middle of it.

Based on these findings, trial organizers should devise recruitment strategies that enhance positive attitudes toward participation in clinical studies. Based on the findings that people should be paid to participate in clinical trials, all trial-related medical procedures including all adverse effect-related interventions should be compensated. The opportunity costs of time spent in the trials should also be compensated.

Trial organizers should also find ways to increase the participation of children in clinical research. At present, parents and guardians are often not willing to permit their children to take part in clinical trials, perhaps because they view the kids as vulnerable and subject to unforeseen risks. In general, trial organizers should inform all segments of society about the benefits of participation.

6. Limitations of the Study

One limitation is that the study focuses only on undergraduates in one university in Kazakhstan. So the findings may not accurately reflect the entire student population in all universities in the country. Also, while this study provides insights into the views of a relatively young student population, a study of the attitudes of the general population toward clinical studies is warranted. Finally, the convenience sampling limits the generalizability of findings even within the university where the study was conducted.

7. Conclusion

Clinical trials are the principal mechanisms for evaluating new treatment options in medicine. Continued development of medical therapeutics requires well-designed clinical trials, which must include a critical mass of representative, dedicated, willing, and fully informed participants.

Recruitment of subjects into clinical trials is essential if we are to innovate in healthcare. This study provides insights into attitudes of undergraduates in Kazakhstan towards clinical trials. Clinical trials aim to provide a high standard of care and to help contribute to increased medical knowledge; but it is still a new concept to many citizens around the world, including Kazakhstanis. To advance our understanding of diseases and to search for new treatment and preventive methods, more opportunities for clinical research will require increasing the pool of study participants.

The factors identified in this exploratory study would aid the design of clinical studies, especially in considering the views of young Kazakhstanis. In that regard, this study is an essential step in understanding the barriers to, and motivation for, participation in clinical trials.

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8. Summary

English: This study was conducted to explore the attitudes of undergraduate students in Kazakhstan toward clinical trials' participation. A 13-item questionnaires survey was administered to 187 university undergraduate students in an English-speaking university in Kazakhstan that is based on the United States model of education. Of the 187 questionnaires administered, 162 were correctly completed and returned. Results of the study suggest that clinical research teams should endeavor to create more awareness that promotes positive attitudes among potential participants. They should also consider cultural differences in clinical trials' recruitment process.

Russian: Eto issledovaniye bylo provedeno dlya izucheniya otnosheniya studentov bakalavriata v Kazakhstane k uchastiyu v klinicheskikh ispytaniyakh. Anketnyy opros iz 13 punktov byl proveden sredi 187 studentov bakalavriata angloyazychnogo universiteta v Kazakhstane, osnovannogo na modeli obrazovaniya Soyedinennykh Shtatov. Iz 187 zapolnennykh anket 162 byli pravil'no zapolneny i vozvrashcheny. Rezul'taty issledovaniya pokazyvayut, chto gruppy klinicheskikh issledovaniy dolzhny stremit'sya k povysheniyu osvedomlennosti, kotoraya sposobstvuyet polozhitel'nomu otnosheniyu sredi potentsial'nykh uchastnikov. Im takzhe sleduyet uchityvat' kul'turnyye razlichiya v protsesse nabora uchastnikov dlya klinicheskikh ispytaniy.

Kazakh: Bul zerttew Qazaqstandağı bakalavrïat stwdentteriniñ klïnïkalıq zerttewlerge qatıswğa degen közqarasın zerttew maqsatında jürgizildi. AQŞ-tıñ bilim berw ülgisine negizdelgen Qazaqstandağı ağılşın tildi wnïversïtettiñ 187 joğarı oqw ornınıñ stwdentterine 13 suraqtan turatın sawalnama jürgizildi. Basqarılğan 187 sawalnamanıñ 162 -si durıs toltırılıp, qaytarıldı. Zerttew nätïjeleri klïnïkalıq zerttew toptarınıñ potencïaldı qatıswşılar arasında jağımdı qarım - qatınastı nasïxattaytın köbirek aqparattandırwğa tırıswı kerek ekenin körsetedi. Olar sonımen qatar klïnïkalıq sınaqtardıñ jaldaw procesinde mädenï ayırmaşılıqtardı eskerwi kerek.

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Appendix

Clinical trials questionnaire

Clinical trial is the use of human participants in medical research for the purpose of developing new vaccines and treatment methods for diseases. Based on your understanding of the above definition, please answer the following questions:

1. Your gender: 1 = Male; 2 = Female

Factors that affect participation in clinical trials:

Factor	1 = Strongly disagree	2 = Disagree	3 = Neutral	4 = Agree	5 = Strongly agree
Children should be able to participate in clinical trials					
Clinical trials are costly for patients to participate					
Clinical trials are done to improve health standards					
Clinical trials are used by pharmaceutical companies only to make money					
Clinical trials should be voluntary					
People should be informed about potential risks of participating in clinical trials					
People should be paid to participate in clinical trials					
People who participate in clinical trials should keep it a secret because of the stigma attached to it (e.g. being used as a guinea pig)					
I am **less likely** to participate in clinical trials if my medical records will be made public					
I am **more likely** to participate in clinical trials if advised to do so by my doctor					
I am **more likely** to participate in clinical trials if it will make or my loved one live longer					
I am **more likely** to participate in clinical trials if doing so will help other patients					
My **greatest concern** in participating in clinical trials is the potential risk to my health					
My greatest concern in participating in clinical trials is that I may not be able to withdraw if I change my mind in the middle of the trial					

THANK YOU FOR YOUR TIME

An Algorithm for Deriving Classical and Generalized Frobenius Numbers: Applications in Tiling and Storing

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Abstract: This paper introduces an effective exact algorithm for deriving classical and generalized Frobenius numbers. It applies the algorithm to solve tiling and storing problems. It also provides empirical arguments in favor of the polynomial-time complexity of the algorithm.

AMS subject classifications: 11D04, 11D07, 11D45, 11Y50

Keywords: Linear Diophantine Equations, Frobenius Number, Generalized Frobenius Numbers, Polynomial-Time Complexity, Tiling

1. Introduction

The classical Frobenius problem (known as the postage-stamp or money-exchange problem) concerns nonnegative integer solutions to the linear equation

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = N, \tag{1.1}$$

where $a_i, i = 1, \dots, n,$ and N are positive integers, $n \geq 2$ and $x_i \in Z_{\geq 0}$. It is also assumed that $a_1 < a_2 < \cdots < a_n$ and $\gcd(\boldsymbol{a} = (a_1, \dots, a_n)^T) \coloneqq \gcd(a_1, \dots, a_n) = 1$. The last condition guarantees the existence of a finite largest integer N, called the Frobenius number $F(\boldsymbol{a}) = f(a_1, \dots, a_n)$, for which the equation in (1.1) has no solutions (see, e.g., Owens, 2003, p.264).

Frobenius numbers have many applications that relate to business, such as solving tiling problems, complexity analysis of the Shell-sort method, Petri nets, partitioning of vector spaces, monomial curves, algebraic geometric codes (Ramírez-Alfonsín, 2005), and stoichiometry determination for mass-spectrometry data (Agarwal, Cazals, & Malod-Dognin, 2012).

The history of this problem was thoroughly surveyed by Ramírez-Alfonsín (2005). If $n = 2$, then $F(\boldsymbol{a}) = a_1a_2 - a_1 - a_2$. This result has been known since 1884. For $n = 3$ the explicit closed form expressions for $F(\boldsymbol{a})$ have been recently obtained by Tripathi (2017). For $n > 3$ no analytical results except lower and upper bounds on $F(\boldsymbol{a})$ are known so far. Using a Turing reduction from the integer knapsack problem, which was considered to be NP-complete, Ramírez-Alfonsín (1996) showed that the Frobenius problem should be NP-hard.

Several not polynomial-time but rather fast algorithms for Frobenius numbers have been developed, including algorithms of Beihoffer, Hendry, Nijenhuis, and Wagon (2005) and Einstein Lichtblau, Strzebonski, and Wagon (2007).

Beck and Robins (2003) introduced a generalized Frobenius number $F_s(\boldsymbol{a})$ (or s-Frobenius number with a positive integer $s \geq 1$) as the largest number that cannot be presented in at least s different ways as a non-negative linear combination of the a_i's. Mathematically

$$F_s(\boldsymbol{a}) = \max\{N \in Z : \#\{\boldsymbol{z} \in Z^n_{\geq 0} : \langle \boldsymbol{a}, \boldsymbol{z} \rangle = N\} < s\}, \tag{1.2}$$

where $\langle \cdot,\cdot \rangle$ stands for the inner product of vectors on R^n. Evidently $F_1(\boldsymbol{a}) \equiv F(\boldsymbol{a})$. Aliev, Fukshansky, and Henk (2011), for $n \geq 2, s \geq 1$, derived the following lower and upper bounds:

$$F_s(\boldsymbol{a}) \geq s^{\frac{1}{n-1}}\left((n-1)! \prod_{i=1}^n a_i\right)^{\frac{1}{n-1}} - \sum_{i=1}^n a_i, \tag{1.3}$$

$$F_s(\boldsymbol{a}) \leq F_1(a) + (s-1)^{\frac{1}{n-1}}\left((n-1)! \prod_{i=1}^n a_i\right)^{\frac{1}{n-1}}. \tag{1.4}$$

Note that some authors define $F_s(\boldsymbol{a})$ as the largest number that cannot be presented in exactly s different ways as a linear combination of the a_i's. This paper uses the definition (1.2).

Section 2 discusses solutions of the equation in (1.1) and the well-known algorithm of Brauer and Shockley (1962). Section 3 describes the proposed algorithm. Section 4 is devoted to computer experiments on $F_s(\boldsymbol{a})$. Section 5 considers problems of tiling and storing goods. Section 6 discusses results and concludes the paper.

2. Preliminaries

The key point of the proposed algorithm is the enumeration of all existing nonnegative integer solutions of the equation in (1.1). A lot of research has been devoted to determining the number of such solutions, called the denumerant. The classical approach is based on the power series expansion of Euler's generating function $\psi(z) = 1/\prod_{i=1}^{n}(1 - z^{a_i})$, but it is not easy to extract the coefficient at z^n in this expansion. A much simpler generating function $\psi(z) = (1 + z^{a_1} + z^{a_2} + \cdots + z^{a_n})^m$ was proposed by Hardy and Littlewood (1966). Using this function, Voinov and Nikulin (1997) developed a simple algorithm that permitted not only explicitly evaluating the denumerant but constructing all existing solutions. Surprisingly, this result during more than 20 years was not mentioned by the community. Mahmoudvand, Hassani, Farzaneh, and Howell (2010) independently derived the same algorithm, but they did not show how to use it for constructing solutions. To enumerate all solutions of the equation in (1.1), the ideas of Voinov and Nikulin (1997) can be used. The generating function for the number $R_N(m,n)$ of compositions of N as in (1.1) on at most $m \leq N$ parts is

$$\psi(z) = (1 + z^{a_1} + z^{a_2} + \cdots + z^{a_n})^m = \sum_{N=0}^{ma_n} R_N(m,n) z^N, \tag{2.1}$$

where

$$R_N(m,n) = \sum_{x_n=0}^{\left[\frac{N}{a_n}\right]} \sum_{x_{n-1}=0}^{\left[\frac{N-a_n x_n}{a_{n-1}}\right]} \cdots \sum_{x_2=0}^{\left[\frac{N-a_n x_n-\cdots-a_3 x_3}{a_2}\right]} \frac{m!}{(m-x_1-\cdots-x_n)!\,x_1!\cdots x_n!} \tag{2.2}$$

if $x_1 + x_2 + \cdots + x_n \leq m$, and $x_1 = (N - a_n x_n - \cdots - a_2 x_2)/a_1$ being a nonnegative integer. Otherwise $R_N(m,n)$ is zero. The sign $[a]$ in this paper stands for the greatest integer part of a. The solutions of the equation in (1.1) are written down as

$$\{a_1^{x_1}, a_2^{x_2}, \dots, a_n^{x_n}\}, \tag{2.3}$$

where $\{x_2, x_3, \dots, x_n\}$ are all sets of summation indices in (2.2). If $m = [N/a_1]$, then formulas (2.2) and (2.3) enumerate all $\mathcal{N}$ existing solutions of the equation in (1.1).

The above algorithm was realized as the R-command "*nlde*" in the R-package "*nilde*" by Pya Arnqvist, Voinov, Makarov, and Voinov (2021). The main advantage of this algorithm is that unlike numerous heuristic and integer linear programming methods, it enumerates all existing solutions of the equation in (1.1). Consider the following instance from Einstein et al. (2007, p. 39):

$$1000000x_1 + 1692802x_2 + 3706550x_3 + 8980199x_4 = 1023045256.$$

The approach used in Einstein et al. (2007) gives only one solution (208,308,55,10). At the same time, commands *library(nilde); nlde(a=c(1000000,1692802,3706550,8980199), n=1023045256)* return

The number of solutions: 3
Solutions:

	sol.1	sol.2	sol.3
x_1	208	570	264
x_2	308	63	105
x_3	55	45	26
x_4	10	20	54

Note that the command "*nlde*" returns "*no solutions*" if there are none.

Lambe (1977) derived a rather tight upper bound on $\mathcal{N}$ as

$$\mathcal{N} \leq \binom{n-1+B}{n-1} \frac{1}{\prod_{i=1}^{n} a_i} = \frac{n(n+1)\cdots(n+B-1)}{B! \prod_{i=1}^{n} a_i}, \tag{2.4}$$

where

$$B = N + \frac{a_1 a_2}{f_2} - 1 + \sum_{i=3}^{n} \left[\frac{a_i f_{i-1}}{2 f_i}\right] \tag{2.5}$$

and f_i is the largest common factor of the set $\{a_1, a_2, \dots, a_i\}, i = 2, \dots, n,$ with $f_n = 1$. From (2.4) one sees that the upper bound for $\mathcal{N}$ is a polynomial in *n* of order *B*.

Brauer and Shockley (1962), Lemma 3, introduced an algorithm that can be extended for calculating the generalized Frobenius numbers as follows:

THEOREM 1. $F_s(a_1, a_2, \dots, a_n) = \max_{1 \leq j \leq a_1 - 1} m_j - a_1,$ (2.6)

where m_j is the least positive integer N congruent to j mod a_1 such that for $n \geq 2$ the Frobenius equation

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n = N, x_i \geq 0, \tag{2.7}$$

has at least $s = 1,2, \dots$ solutions in nonnegative integers.

Proof. Each residue class modulo a_1 contains numbers representable by a linear combination $a_2 x_2 + a_3 x_3 + \cdots + a_n x_n, x_i \geq 0,$ (Brauer & Shockley, 1962, p. 217). In each of these residue classes, select the smallest representable number. Let m_j be the maximum of these numbers. From this it follows that $m_j - a_1$ is not representable by $a_1, \dots, a_n$ in nonnegative integers for any $1 \leq j \leq a_1$. At the same time, any *N* greater than each $m_j - a_1$ congruent to *j* mod a_1 is representable by $a_1, \dots, a_n$ and therefore the equation in (2.7) has at least $s = 1,2, \dots$ solutions.

3. The Proposed Algorithm

The ability to get explicitly all existing nonnegative integer solutions of any linear Diophantine equation permits one to create the following exact algorithm:

Step 1. (Initialization). Set the desired values of $s = 1,2, \dots$ and $\boldsymbol{a} = (a_1, \dots, a_n)^T, n \geq 2$.
Step 2. (Sub problem solution). For every $1 \leq j \leq a_1$ and $k \in Z_{\geq 0}$, define the minimal integer m_j for which the equation $a_1 x_1 + \cdots + a_n x_n = j + k a_1$ has at least *s* solutions.
Step 3. (Frobenius number calculation). Compute $F_s(a_1, a_2, \dots, a_n) = \max_{1 \leq j \leq a_1 - 1} m_j - a_1$.

This algorithm was realized as the R-script provided in the Appendix.

Remark 1. Existence of $F(\boldsymbol{a})$ if $\gcd(\boldsymbol{a}) = 1$ means that $F(\boldsymbol{a})$ is finite. To find $F(\boldsymbol{a})$ by the above algorithm, one has to solve the finite number of Frobenius equations (2.7), where the number of solutions of every equation is bounded above by a polynomial (Lambe, 1977). Assuming that any Frobenius equation can be solved in polynomial time, as was empirically shown in Voinov (2017), we may conclude that the finite sequence of solutions will also be solved in polynomial time.

4. Computer Experiments

To illustrate the proposed algorithm, consider the simplest classical example of Chicken McNuggets. McDonald's restaurants in the USA sell nuggets in packages of 6,9, or 20. Using the script for $s = 1,2,3,4$ one obtains $F_1(\boldsymbol{a}) = 43, F_2(\boldsymbol{a}) = 61, F_3(\boldsymbol{a}) = 79, F_4(\boldsymbol{a}) = 97$. From this it follows that 43 nuggets cannot be sold as a combination of 6,9, or 20; for 61 nuggets, there is only one combination; two combinations for 79; and three for 97. The command *nlde(a=c(6,9,20),*$F_s(\boldsymbol{a})$) gives those combinations for s = 2,3,4 explicitly as: (2,1,2), {(5,1,2), (2,3,2)}, and {(8,1,2), (5,3,2), (2,5,2)} respectively.

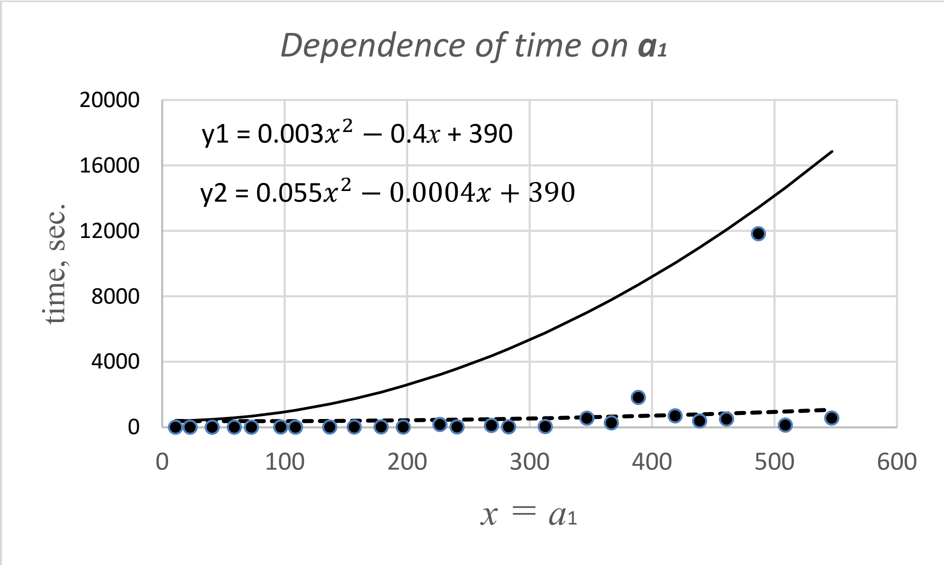
The next example illustrates the algorithm for inputs that are such that desired results cannot be easily obtained in the mind. Let a=c(113,127,157) (see Tripathi, 2017, p. 377). For $s = 1,2,3,4$ one gets $F_1(\boldsymbol{a}) = 2554, F_2(\boldsymbol{a}) = 3697, F_3(\boldsymbol{a}) = 4205, F_4(\boldsymbol{a}) = 4840$. Alternatively, one can obtain the results for $F_2(\boldsymbol{a}), F_3(\boldsymbol{a}), F_4(\boldsymbol{a})$ by the following direct procedure. Using the command *nlde(a=c(113,127,157),N*), find the largest integers *N* for all solutions of the equation (2.7) if $F_1(\boldsymbol{a}) \leq N \leq Ub_s$ such that the number of solutions will be exactly 1,2, or 3. Evidently, those numbers will be 3697, 4205, and 4840 as above; but the computing time, as compared to usage of the R-script, is 3-10 times greater. It has to be noted that the application of the script confirms the correctness of all numerical results for the classical Frobenius numbers published in Tripathi.

The proposed algorithm works also for larger dimensions and larger input values. For a numerical example with $\mathrm{a} = \mathrm{c}(1000,1476,3764,4864,4871,7773)$ from the paper by Beihoffer et al. (2005, p. 11), the values of $F_1(\boldsymbol{a}), F_2(\boldsymbol{a}), F_3(\boldsymbol{a}), F_4(\boldsymbol{a})$ are 47350, 50302, 52302, and 53778 respectively. The value of $F_1(\boldsymbol{a})$ coincides with that obtained in Beihoffer et al. by a different method.

Einstein et al. (2007, p. 60) posed the following algorithmic question: "Suppose $n = 4$. Is there an implementable algorithm that is polynomial-time in the worst case?" In an attempt to answer this question, consider the next two experiments based on the algorithm of Section 3.

(a) The first one uses 25 non-random distinct consecutive 4-tuples $T_i, i = 1,2,\ldots,25$, formed from all 100 prime numbers in the interval [11, 569]: $T_1 = (11,13,17,19), T_2 = (23,29,31,37),\ldots,T_{25} = (547,557,563,569)$. Since $\gcd(T_i) = 1, i = 1,2,\ldots,25$, the Frobenius numbers $F(T_i)$ exist and the corresponding computing times can be defined using the R-script in the Appendix. Figure 1 shows the results of the computation.

Figure 1
Dependence of Time on Parameter a_1

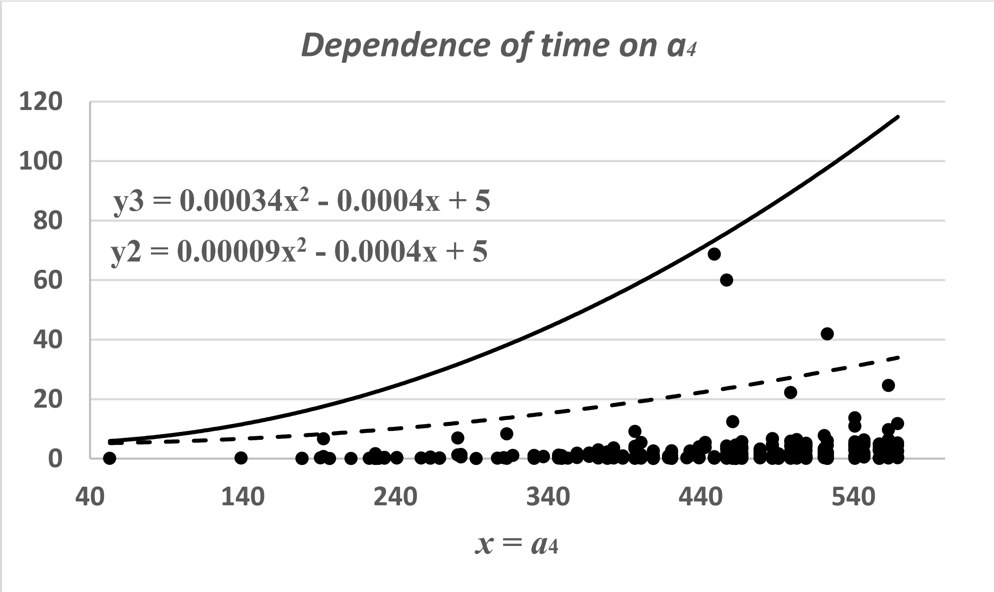


Black circles are computing times in seconds. The polynomial y2 (solid curve) bounds above all measurements. The polynomial y1 (dashed line) bounds above 23 measurements out of 25.

From Figure 1, one sees that the measurements can be divided onto two groups. One (92% of measurements) is bounded by a polynomial with a rather small (0.003) coefficient at x^2. The second group, of 2 measurements, can be considered as a set of outliers (rare exceptions). To investigate this interesting property more thoroughly, the following simulation experiment was performed.

(b) 275 4-tuples of prime numbers in the interval [11, 569] were randomly generated. Figure 2 shows the results of the computation times.

Figure 2
Dependence of Time on Parameter a4



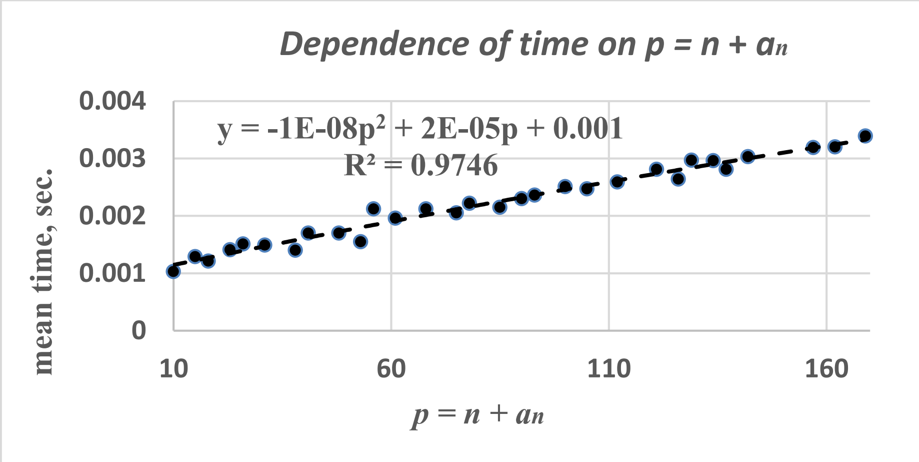
Black circles are computing times in seconds. The polynomial y3 (solid line) bounds above all measurements. The polynomial y2 (dashed line) bounds above 272 (99%) measurements out of 275.

From Figure 2, one sees that random 4-tuples confirm the structure of measurements observed in Figure 1. The difference in the measure of the tuple size, a_1 versus a_4, is not essential, since a Frobenius number does not depend on the way of ordering elements of a vector $\boldsymbol{a} = (a_1, a_2, a_3, a_4)^T$.

The most important conclusion from the experiments in (a) and (b) is that time measurements are bounded above by a polynomial of order 2. This fact can be considered as evidence in favor of a positive answer to the question of Einstein et al. (2007). Since *n* is fixed, one may conclude that the Frobenius problem is solvable in pseudo-polynomial time.

The next experiment investigates the dependence of computing time on the number *n* of elements in a vector $\boldsymbol{a} = (a_1, \dots, a_n)^T$ and the magnitude of a_n. Let $\boldsymbol{a} = (a_1, a_2, \dots, a_{32})^T$ be the set of all 32 prime numbers in the interval [3, 137]. For all subsets $A_n = \{a_1, a_2, \dots, a_n\}, n = 3,4, \dots, 32$, the Frobenius numbers $F_n(A_n)$ and the mean computing times for 1500 runs for every n on a standard PC (Intel® (Core(™) i7-2600 CPU@3.40 GHz, RAM 6.00GB) were obtained. Figure 3 shows the dependence of the mean times on the parameter $p = n + a_n$.

Figure 3
Dependence of time on parameter p



Full circles present the mean times computed with accuracy less than 10%.
Fitted dependence is shown by the dashed line.

From Figure 3 one sees that the computing time for all 32 measurements is bounded above by a polynomial of order 2. Section 6 discusses implications of this result.

5. Applications in Tiling and Storing

By tiling, mathematicians mean a way of arranging small n-dimensional rectangles (e.g., tiles or bricks) so that they entirely fill the given n-dimensional rectangle without overlapping. The 2-dimensional case is used in construction for covering roofs, floors, walls, etc. The 3-dimensional one can be used for storing goods in warehouses.

Ramírez-Alfonsín (2018) used the Frobenius number "to show that a n-dimensional rectangle $A = (a_1 \times a_2 \times \cdots \times a_n)$ can be tiled with some set of bricks if the sides $a_i, i = 1, \dots, n$, are larger than a certain" integer numerical value. Let $x_j^i, j = 1, \dots, n, i = 1, \dots, n+1$, be a positive integer and $\{i_1 < \cdots < i_{k+1}\} \in \{1, \dots, n+1\}, 1 \le k \le n$. Having defined the set

$$X_k(i_1, \dots, i_{k+1}) = \{(x_k^{i_1} \cdots x_k^{i_k}), (x_k^{i_1} \cdots x_k^{i_{k-1}} x_k^{i_{k+1}}), \dots, (x_k^{i_1} x_k^{i_3} \cdots x_k^{i_{k+1}}), (x_k^{i_2} \cdots x_k^{i_{k+1}})\}, \quad (5.1)$$

Ramírez-Alfonsín (p. 3) proved the following:

THEOREM 2. *Let* $x_j^i \ge 2$ *be an integer for each* $j = 1, \dots, n$ *and each* $i = 1, \dots, n+1, n \ge 1$. *Suppose that* $\gcd(X_k(i_1, \dots, i_{k+1})) = 1$ *for any* $\{i_1 < \cdots < i_{k+1}\} \in \{1, \dots, n+1\}, 1 \le k \le n$, *and let*

$$g_n = \max_{1 \le k \le n} \left\{F(X_k(i_1, \dots, i_{k+1})) \middle| \{i_1, \dots, i_{k+1}\} \in \{1, \dots, n+1\}\right\}. \quad (5.2)$$

Then $A = (a_1 \times \cdots \times a_n)$ *can be tiled with bricks* $X^i = (x_1^i \times \cdots \times x_n^i), i = 1, \dots, n+1$ *if* $a_j > g_n$ *for all* $1 \le j \le n$.

Example 1.
Let $n = 2$, $X^1 = (x_1^1 \times x_2^1) = (p \times q) = (6 \times 4)$, and $X^2 = (x_1^2 \times x_2^2) = (r \times s) = (5 \times 7)$. This classical example, posed in the 1991 William Mowell Putnam Examination, was generalized by Ramírez-Alfonsín (2018, p. 2) who included in consideration the third tile $X^3 = (x_1^3 \times x_2^3) = (s \times r) = (7 \times 5)$, which is the rotated second one. In accordance with the definition in (5.1) for $k = 1$ (i.e. $(i_1 < i_2) \in \{1,2,3\}$), we have:

$$X_1(1{,}2) = \{x_1^1, x_1^2\} = \{p, r\} = (6{,}5), X_1(1{,}3) = \{x_1^1, x_1^3\} = \{p, s\} = (6{,}7),$$
$$X_1(2{,}3) = \{x_1^2, x_1^3\} = \{r, s\} = (5{,}7). \quad (5.3)$$

For $k = 2$ (i.e. $(i_1 < i_2 < i_3) \in \{1,2,3\}$)

$$X_2(1{,}2{,}3) = \{x_2^1 x_2^2, x_2^1 x_2^3, x_2^2 x_2^3\} = \{qs, qr, sr\} = (28{,}20{,}35).$$

(5.4)

For $X_1(i_1, i_2)$, the Frobenius numbers $F(i_1, i_2)$ are computed by the formula $F(i_1, i_2) = i_1 i_2 - i_1 - i_2$. For $F(i_1, i_2, i_3)$, Ramírez-Alfonsín (2018) used the well-known results of Brauer and Shockley (1962) that apply in this case. Note that if $n = 3$, then $F(i_1, i_2, i_3)$ can be computed also by using formulas of Tripathi (2017), but it is much simpler to use the R-script in the Appendix. From $\gcd(6{,}5) = \gcd(6{,}7) = \gcd(5{,}7) = \gcd(28{,}20{,}35) = 1$, it follows that all corresponding Frobenius numbers exist and that they are: $F(6{,}5) = 19, F(6{,}7) = 29, F(5{,}7) = 23,$ and $F(28{,}20{,}35) = 197$. Then (5.2) gives $g_2 = \max(19{,}29{,}23{,}197) = 197$. Thus the rectangle area $(a_1 \times a_2)$ can be tiled with the above three tiles of squares 24, 35, and 35 respectively if $a_j \geq 198, j = 1{,}2$. To answer the question "How many tiles of each kind should be used?" we have to solve the equation $24s_1 + 35s_2 + 35s_3 = 198^2 = 39{,}204$. The implementation of the R-command *nlde(a=c(24,35,35),39204)* gives 26,555 solutions. This huge number of solutions seems to be useless, but an application of the optimization may reduce it dramatically. Assume, e.g., that mounting costs for those tiles are 5,13, and 13 respectively, and one wants to define solutions that will minimize the total cost. Calculations show that in this case there are only 13 optimal solutions of the total cost of 8236 that require 1616 tiles of size 6×4 and 12 tiles of size 5×7 or 7×5 in any combination.

Note that by the date the problem of defining exact arranging schemes is not solved.

Example 2.

Let $n = 3$, $X^1 = (x_1^1 \times x_2^1 \times x_3^1) = (l \times m \times n) = (2 \times 3 \times 5)$, $X^2 = (x_1^2 \times x_2^2 \times x_3^2) = (o \times p \times q) = (3 \times 5 \times 7), X^3 = (x_1^3 \times x_2^3 \times x_3^3) = (r \times s \times t) = (5 \times 7 \times 2)$, and $X^4 = (x_1^4 \times x_2^4 \times x_3^4) = (u \times v \times w) = (7 \times 2 \times 3)$. In accordance with the definition in (5.1)

for $k = 1$ (i.e. $(i_1 < i_2) \in \{1,2,3,4\}$) we have: $X_1(1{,}2) = \{x_1^1, x_1^2\} = (l, o) = (2{,}3), X_1(1{,}3) = \{x_1^1, x_1^3\} = (l, r) == (2{,}5), X_1(1{,}4) = \{x_1^1, x_1^4\} = (l, u) = (2{,}7), X_1(2{,}3) = \{x_1^2, x_1^3\} = (o, r) = (3{,}5), X_1(2{,}4) = \{x_1^2, x_1^4\} = (o, u) = (3{,}7)$, and $X_1(3{,}4) = \{x_1^3, x_1^4\} = (r, u) = (5{,}7)$.

For $k = 2$ (i.e. $(i_1 < i_2 < i_3) \in \{1,2,3,4\}$) $X_2(1{,}2{,}3) = \{x_2^1 x_2^2, x_2^1 x_2^3, x_2^2 x_2^3\} = (mp, ms, ps) = (15{,}21{,}35), X_2(1{,}2{,}4) = \{x_2^1 x_2^2, x_2^1 x_2^4, x_2^2 x_2^4\} = (mp, mv, pv) = (15{,}6{,}10)$, $X_2(1{,}3{,}4) = \{x_2^1 x_2^3, x_2^1 x_2^4, x_2^3 x_2^4\} = (ms, mv, sv) = (21{,}6{,}14)$, and $X_2(2{,}3{,}4) = \{x_2^2 x_2^3, x_2^2 x_2^4, x_2^3 x_2^4\} = (ps, pv, sv) = (35{,}10{,}14)$.

For $k = 3$ (i.e. $(i_1 < i_2 < i_3 < i_4) \in \{1,2,3,4\}$) $=X_3(1{,}2{,}3{,}4) = \{x_3^1 x_3^2 x_3^3, x_3^1 x_3^2 x_3^4, x_3^1 x_3^3 x_3^4, x_3^2 x_3^3 x_3^4\} = (nqt, nqw, ntw, qtw) = (70{,}105{,}30{,}42)$. Computing all Frobenius numbers involved by our R-script, we get $g_3 = 383$. From (5.2) it follows that the above four bricks can be tiled if $a_1 \times a_2 \times a_3 \geq 384^3$. If, e.g., sizes of those bricks are given in centimeters, then one will need about 57 cubic meters of free space in a warehouse.

6. Discussion and Conclusions

Any algorithm belongs to the class P if its computing time is bounded above by a polynomial of the instant "size" of a problem. In theory this "size" is considered as a single parameter "which reflects the number of symbols that would be required to describe the instance in a 'reasonable' and 'concise' manner" (Garey & Johnson, 1978, p. 500). For the Frobenius problem, that "size" can be described, e.g., by a parameter $p = n + a_n$ used in this research (Garey & Johnson, 1979, p. 20). From the above and from Figure 3, it follows that the computing time of the algorithm in (2.6) on a deterministic computer is bounded above by a polynomial of order 2, and hence the Frobenius problem belongs to the class P. One could expect this result, because the finite number of solutions of the equation in (2.7), used in deriving the algorithm, is due to the fact that the formula in (2.4) is bounded above by a polynomial.

Using the assumption that the integer knapsack problem is NP-complete, Ramírez-Alfonsín (1996) has proved the NP-hardness of the Frobenius problem. He proved this by a polynomial time Turing reduction from the integer knapsack problem. Our main result contradicts to this conclusion because it shows that the Frobenius problem is not only NP-hard but, being solvable in polynomial time, is even not NP at all. Moreover, as Stephen Cook noted, "if we have a polynomial time reduction from one problem to another, this ensures that any polynomial-time algorithm for the second problem can be converted into a corresponding polynomial-time algorithm for the first problem" (see Garey and Johnson, 1979, p. 13). Thus, from our results, it follows that integer knapsack problems belong to the class P. This was also empirically shown in Voinov (2017).

From all the above it can be concluded that the empirical results of this research are in favor of the fundamental equality P=NP. But this conclusion cannot be considered as a strong final one for the equality P=NP because: (a) one should still check polynomial-time complexity of the algorithm for larger values of the parameter $p = n + a_n$ that needs a more powerful computer, and (b) polynomial-time complexity of the algorithm in (2.2) should be justified theoretically.

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7. Summary

English: An exact polynomial-time algorithm for classical and generalized Frobenius numbers' derivation is introduced. The main mathematical result can be considered as an empirical argument in favor of the fundamental equality P=NP. Applications of the algorithm for tiling and storing goods are discussed.

Russian: Предложен точный полиномиальный алгоритм для вычисления классических и обобщенных чисел Фробениуса. Главный математический результат может рассматриваться как эмпирический аргумент в пользу фундаментального равенства P=NP. Обсуждаются применения алгоритма для мозаичного размещения и хранения товаров.

Kazakh: Нақты көпмүшелік алгоритм классикалық және жалпыланған Фробениус санын есептеу үшін ұсынылады. Негізгі математикалық нәтиже P = NP фундаменталды теңдік

пайдасына эмпирикалық дәлел ретінде қаралуы мүмкін. Мозаикалық орналастыру және тауарларды сақтау алгоритмін қолдану талқыланады.

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Appendix

R-Code

```
library(nilde)
c<-proc.time()
s<-1 #2,3,...
a1<-c(6,9,20)
a2<-a1[1]-1
a3<-a1[1]
Sol<-rep(0,a2)
i=1
for(i in 1:a2){
k<-0
while(k>=0){
b1<-nlde(a=a1,i+k*a3)
b2<-b1$p.n
if(b2>s-1){
Sol[i]<-(i+k*a3)
break}
k=k+1;};i=i+1;}
F<-max(Sol)-a3
F#43
j<-proc.time()
print(j-c)
```

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